

# **Predicting Deformation and Strain Behavior in Circuit Board Bend Testing**

**by Ronen Aniti**

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**May 2012**

**prepared by**

**Science and Engineering Apprenticeship Program  
George Washington University  
2121 Eye St NW  
Washington, DC 20052**

**under contract**

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14. ABSTRACT This report presents mathematical and finite element (FE) models that were used to predict the deformation and strain levels of a circuit board under a variety of four-point monotonic bend test configurations. The derivations of the mathematical models for deformation and strain are examined, and an equation modeling the strain rate of the board as it is bending is presented. The two and three-dimensional (2-D and 3-D) FE models are discussed and validated and the differences in deformation and strain levels between the models are accounted for. A convergence study was conducted to determine the optimal number of elements to use in these models and the results are also presented. In addition, key trends in the computed deformation and strain data across various test configurations are identified.					
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## 1. Introduction

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Researchers at the U.S. Army Research Laboratory widely employ bend testing to determine the reliability and survivability of printed circuit boards and other electronic components under various load conditions. In conjunction with vibratory test methods, researchers are able to simulate realistic battlefield environments consisting of a wide range of vibratory loads, to accurately test for faults in the behavior of the components that control the devices that are necessary for modern warfare.

This report discusses the modeled deformation and strain behavior of an FR4 circuit board with no mounted electronic packaging as it is subjected to various ASTM D7262 four point monotonic bend test configurations. Basic geometric principles were used to model this behavior with respect to three bend test variables: (1) board deflection distance, (2) inner contact separation distance, and (3) outer contact separation distance. These variables (shown in figure 1) are adjusted by the researcher prior to the test.

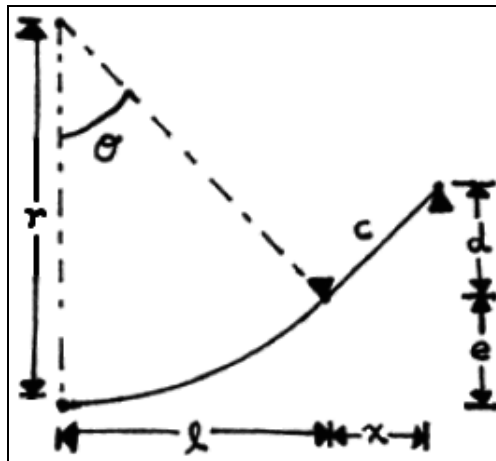


Figure 1. Sketch of half the circuit board.

Once this was complete, finite element (FE) simulations of the board under bending were run in two and three-dimensions (2-D and 3-D). However, these models, unlike the geometric models, took into consideration both the material properties and thickness of the board. The deformation and strain data provided by these models ended up both confirming the relative accuracy of the geometric models and providing further insight on the behavior of the circuit board under bending. In particular, the more realistic FE models rendered the local deformation on the surface of the board around the contact points whereas the one dimensional geometric models did not. This led to a greater understanding of the effects of this local deformation on the overall deformation and strain levels subjected to the board.

Because it was of primary concern in this particular study to examine the effects of changing the outer contact separation distance and board deflection distance within a narrow range while keeping inner contact separation distance constant, deformation and strain levels were plotted for five contact separation and board deflection distances.

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## 2. Geometric Models

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The first issue in modeling the board deformation and strain behavior geometrically is predicting the way in which the board will bend. Under ideal ASTM D6272-10 monotonic bend test conditions; the radius of curvature of the board between the two inner contact points remains constant during bending.<sup>1</sup> For this experiment, the board was modeled geometrically according to two different idealizations: (1) that the radius of curvature remained constant between the outer contacts during bending, and (2) that the radius of curvature between the two inner contacts remained constant, but that the board was straight between the inner and outer contacts. Both of these idealizations are consistent with ASTM D6272-10; however, they differ in the assumed behavior between the inner and outer contacts and allow for a relatively simple closed form solution to the geometric configuration. Realistically, the board most likely behaves somewhere in between the assumed idealizations. All the variables used in the derivations of the models are defined in table 1 below.

Table 1. Definitions of variables used in flexural calculations.

$\varepsilon$	Board Surface Strain
$e$	Deformation of Center of Board
$x$	Outer Contact Separation
$l$	Inner Contact Separation
$d$	Board Deflection
$r$	Radius of Curvature at Center of Board
$T$	Board Thickness
$A$	Amplitude of Vibration
$\omega$	Frequency of Vibration
$t$	Time

---

<sup>1</sup>ASTM Standard D6272, 2010, "Test Method for Flexural Properties of Unreinforced and Reinforced Plastics and Electrical Insulating Materials by Four-Point Bending," ASTM International, West Conshohocken, PA, 2003, DOI: 10.1520/D6272-10, [www.astm.org](http://www.astm.org).

For the first idealization,  $r$  at the inner and outer contacts is related to  $x$ ,  $l$ , and  $d$  with the Pythagorean Theorem. Since  $r$  is the same at both contact points, equations 1 and 2 are set equal to each other and to yield an equation for board deformation.

$$r^2 = (d + e - r)^2 + (l + x)^2 . \quad (1)$$

$$r^2 = (r - e)^2 + l^2 . \quad (2)$$

$$(r - e)^2 + l^2 = (d + e - r)^2 + (l + x)^2 . \quad (3)$$

$$e = - \frac{2lx + d^2 + x^2 + \sqrt{d^4 + 4d^2l^2 + 4d^2lx + 2d^2x^2 + 4l^2x^2 + 4lx^3 + x^4}}{2d} . \quad (4)$$

Also for this idealization, board surface strain can be solved for using the following definition of strain:

$$\varepsilon = \frac{1}{2} \times \frac{T}{r} . \quad (5)$$

Therefore,

$$\varepsilon = \frac{1}{2} \times \frac{T}{\sqrt{(d+e-r)^2 + (l+x)^2}} . \quad (6)$$

Also significant for bend testing computations is an equation for strain rate on the surface of the board. Strain rate ( $\dot{\varepsilon}$ ) is defined as:

$$\dot{\varepsilon} = \frac{\partial \varepsilon}{\partial d} \times \dot{e} . \quad (7)$$

In this equation an alternate definition for  $e$  is used that takes into account the amplitude of vibration, frequency of vibration and time:

$$e = A \sin(\omega t) . \quad (8)$$

Therefore the strain rate for this idealization is

$$\dot{\varepsilon} = \frac{T \times A \omega (d-r+e) \cos(\omega t)}{2[(d-r+e)^2 + (l+x)^2]^{\frac{3}{2}}} . \quad (9)$$

As for the second idealization where section  $c$  is straight, the following trigonometric relationships were determined by incorporating the Pythagorean Theorem. These led to an equation for  $r$  shown in equation 17.

$$l = r \sin \theta . \quad (10)$$

$$x = c \cos \theta . \quad (11)$$

$$d = c \sin \theta . \quad (12)$$

$$d = \frac{c \times l}{r}. \quad (13)$$

$$c = \sqrt{x^2 + d^2}. \quad (14)$$

$$\cos \theta = \frac{x}{\sqrt{x^2 + d^2}}. \quad (15)$$

$$d = \frac{\sqrt{x^2 + d^2} \times l}{r}. \quad (16)$$

$$r = \frac{l \times \sqrt{x^2 + d^2}}{d}. \quad (17)$$

Board deformation was then related to  $r$  in equation 17 and solved for by substitution of the above equations to the right side of the equation.

$$e = r - r \cos \theta. \quad (18)$$

$$e = r - r \times \frac{x}{\sqrt{x^2 + d^2}}. \quad (19)$$

$$e = r \times \left(1 - \frac{x}{\sqrt{x^2 + d^2}}\right). \quad (20)$$

$$e = \frac{l \times \sqrt{x^2 + d^2}}{d} \times \left(1 - \frac{x}{\sqrt{x^2 + d^2}}\right). \quad (21)$$

$$e = \frac{l \times \sqrt{x^2 + d^2}}{d} - \frac{l \times x}{d}. \quad (22)$$

$$e = \frac{l}{d} \times (\sqrt{x^2 + d^2} - x). \quad (23)$$

Surface strain is solved for using equation 5:

$$\varepsilon = \frac{1}{2} \times \frac{T \times d}{l \times \sqrt{x^2 + d^2}}. \quad (24)$$

Strain rate is solved for by using equations 7 and 8 in the same manner as before.

$$\dot{\varepsilon} = \frac{T x^2 A \omega \cos(\omega t)}{2l(d^2 + x^2)^{\frac{3}{2}}}. \quad (25)$$

### 3. The FE Models

With these derived equations, the researcher is more able to estimate the effects on the deformation and strain behavior from a change in any of the initial test variables. The researcher will, for example, be able to adjust a test so that the board is met with a specific deformation and strain profile. However, how is it possible to determine the accuracy of these models if there is nothing to compare them to? To gain a better assurance in the derived equations, the equations were validated against 2- and 3-D FE simulations of the bend test.

The models were constructed in ANSYS version 10 using orthotropic material properties taken from appendix B of IPC/JEDEC 9704 for FR4.<sup>2</sup> Due to symmetric geometry and bounding conditions, one half of the circuit board was modeled. The board thickness was set to 1/4th of an inch and the inner contact separation distance was set to 3.477 inches in order to match the specifications of the equipment on hand. Before running the FE simulation, a convergence study was run to determine the number of elements to use. In this study the board was modeled in 2-D with the specifications listed in table 2, and a 0.1045 in vertical deflection distance, about half of the maximum deflection distance permitted by the equipment used to run this test.

Table 2. Material properties of FR4 used in FE models.

FR-4 Material Properties Used in FEA Model	Value (GPa)
Young's Modulus ( $E_x$ )	19
Young's Modulus ( $E_z$ )	19
Young's Modulus ( $E_y$ )	3.4
Shear Modulus ( $G_{xz}$ )	6.89
Shear Modulus ( $G_{xy}$ )	1.38
Shear Modulus ( $G_{yz}$ )	1.38
Poisson's Ratio ( $PR_{xz}$ )	0.16
Poisson's Ratio ( $PR_{xy}$ )	0.25
Poisson's Ratio ( $PR_{yz}$ )	0.25

The results of the test are shown in figure 2 below.

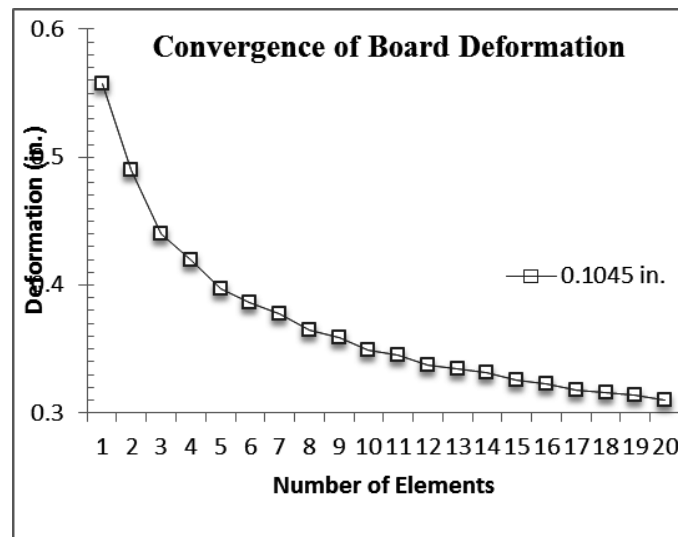


Figure 2. Convergence plot of board deformation.

<sup>2</sup>IPC/JEDEC 9702. Monotonic Bend Characterization of Board Level Interconnects, June, 2004.

In determining the number of elements to use in the validation of the FE models it was important to choose a number that would produce relatively accurate results in as little time as possible. In order to do this, the percent differences between subsequent deformation values were calculated (see table 3). It was decided that a percent difference of ~5% would satisfy this condition, so five elements were used in the validation of the FE models.

Table 3. Percent differences in subsequent deformation values.

No. of Elements	Deformation	Percent Difference
1	0.55746874	
2	0.49011365	12.08%
3	0.44022665	10.18%
4	0.41979439	4.64%
5	0.39682947	5.47%
6	0.38605562	2.71%
7	0.37770322	2.16%
8	0.36463678	3.46%
9	0.35888738	1.58%
10	0.34950194	2.62%
11	0.34512509	1.25%
12	0.33790021	2.09%
13	0.33432656	1.06%
14	0.33125853	0.92%
15	0.325631	1.70%
16	0.32299228	0.81%
17	0.31835312	1.44%
18	0.31608611	0.71%
19	0.31406893	0.64%
20	0.31014461	1.25%

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## 4. Results and Discussion

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The results of the validation of the equations are shown in figure 3 below. In these graphs the average values for deformation and strain are compared to the 2- and 3-D FE models across 10 horizontal contact separation displacements. The difference in conditions between the two graphs of each given deflection is apparent.

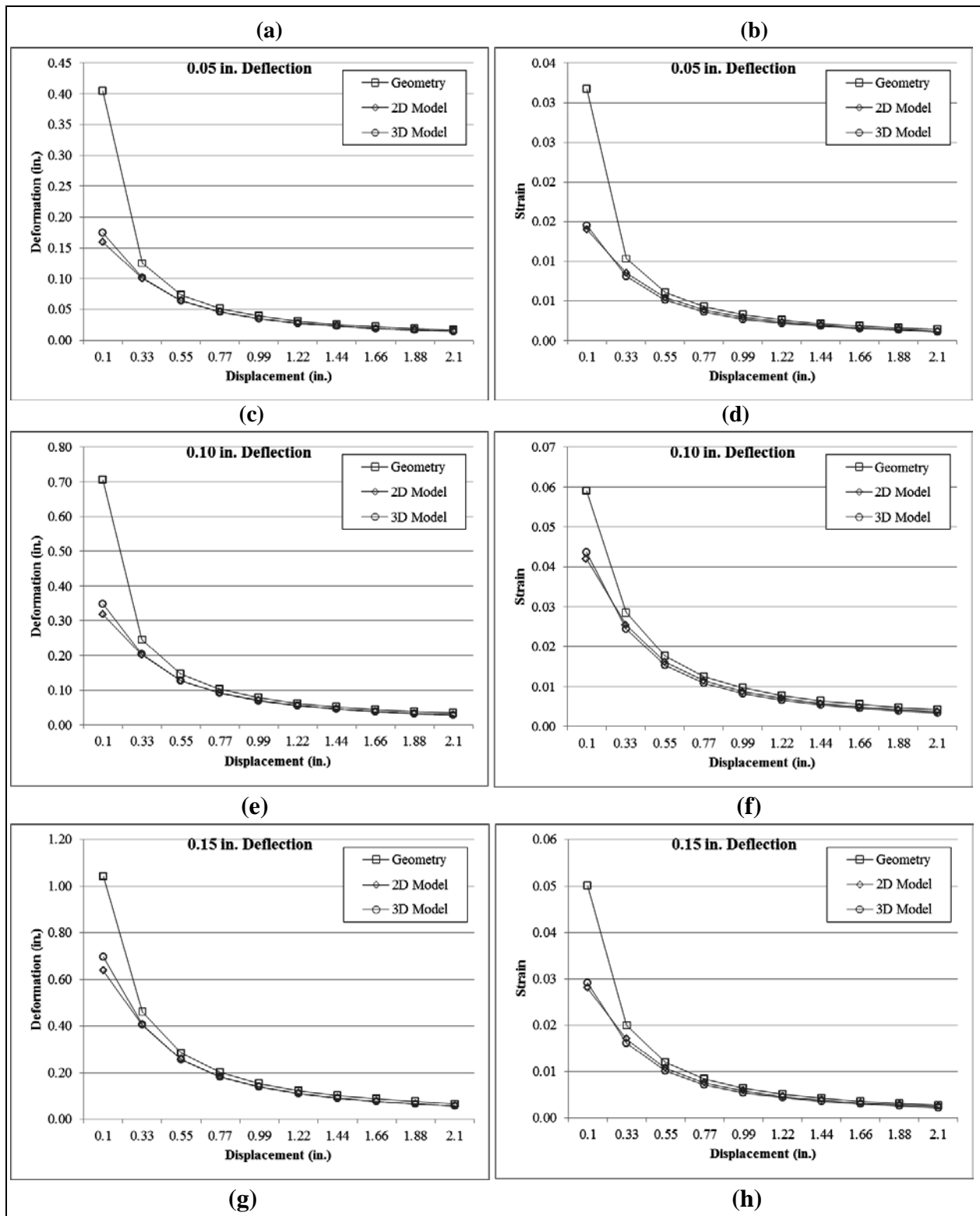


Figure 3. Deformation and strain plots for multiple board deflection distances.

A couple trends in the board deformation and strain behavior are evident from the data presented above. Firstly, the geometric model tends to predict greater deformation and strain values than the FE models with a more dramatic difference at lower contact separation displacements. Secondly, both the computed deformation and strain values from the geometric models and the FE models tend to get closer together at higher board deflection distances. That the FE models tend to produce lower deformation and strain values is most likely the cause of the local deformation that occurs about the contact points. However, this effect is greatly mitigated as the horizontal separation displacement and board deflection increase. Also noteworthy is that the 2- and 3-D FE models produce nearly identical output.

It would seem logical to use the 3-D model to aid in all deformation and strain data predictions, since it is the most realistic. However, since time is a primary determinant in which model is selected to be used, the researcher is almost as well off using the geometric equations, except at horizontal contact separation displacement of less than a third of an inch, as they require virtually no computation time. For these very low separation displacements, the researcher is best off using the 2-D FE model as it is significantly quicker than running the same test in 3-D and produces nearly identical results.



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